The Saturated Bayesian

Break Detection in Panel Data with Short Time Horizons

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Introduction

- **Macro and/or Environmental Data:** Often large N and small T
- **Interventions as Breaks:** interventions can appear as positive or negative breaks at unknown times.
- **Evaluation Focus:** Traditional policy evaluations focus on the effects of single, known policies, contributing to uncertainty.
- **Policy Tool Combination:** Policymakers use various policies (and mixes) in pursuit of their goals, but effectiveness remains uncertain.
- **Delayed Effect:** The potential delay between intervention and its effects requires flexibility, as these two often diverge.

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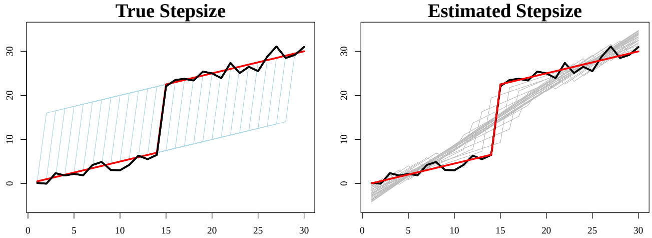
- **Problem:** Macroeconomic as well as climate data are often weak-sense non-stationary. Besides detecting breaks, one might be interested in:
  1. **the partial effects**
  2. **the size of the break itself**

- Regime switching models too restrictive in state transitions.

- Approach: **Step Indicator Saturation (SIS)** to detect breaks



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- Frequentist framework developed in Castle et al. (2015)
  - "General to specific" a.k.a. "Gets" as our **performance benchmark**
  - Extended to panels in Pretis and Schwarz (2022)
- We propose a flexible **Bayesian break detection model** with
  - strong detection quality in various settings,
  - natural break-time uncertainty quantification,
  - intuitively interpretable prior parameters,
  - and an outlier-robust estimation strategy.
- We showcase our approach with
  - a **simulation**, study benchmarking with "Gets",
  - a **replication** of break detection in transport emissions (Koch et al. 2022),
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Setup

Given observations $(Y_{1,1}, \mathbf{X}_{1,1}) \dots (Y_{N,T}, \mathbf{X}_{N,T})$, the model is

$$y_{i,t} = \mathbf{x}'_{i,t}\beta + \sum_{j=3}^{T-1} \mathbb{1}_{\{j \le t\}} \delta_{i,t} \gamma_{i,t} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim \pi_\varepsilon \tag{1}$$

or in matrix notation

$$y = \mathbf{X}\beta + \mathbf{Z}\Delta\gamma + \varepsilon, \quad \varepsilon \sim \pi_{\varepsilon\{NT\}}$$

where $\mathbf{X} \in \mathbb{R}^{NT \times p}$ contains e.g. fixed effects or external regressors.

Construction of $\mathbf{Z}$

$$\mathbf{Z} = \underbrace{\begin{bmatrix} \mathbf{z} & & 0 \\ & \ddots & \\ 0 & & \mathbf{z} \end{bmatrix}}_{NT \times N(T-3)} \quad \text{with} \quad \mathbf{z} = \underbrace{\begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 1 & \ddots & \vdots \\ \vdots & \ddots & 0 \\ 1 & \cdots & 1 \\ 1 & \cdots & 1 \end{bmatrix}}_{T \times (T-3)}$$

- **$\mathbf{Z}$ is block diagonal** with N blocks each being **a binary matrix $\mathbf{z}$ collecting step-shifts**, which is $T \times (T-3)$.

Model Selection Problem

Given observations $(Y_{1,1}, \mathbf{X}_{1,1}) \dots (Y_{N,T}, \mathbf{X}_{N,T})$, the model is

$$y = \mathbf{X}\beta + \mathbf{Z}\Delta\gamma + \varepsilon, \quad \varepsilon \sim \pi_{\varepsilon\{NT\}}$$

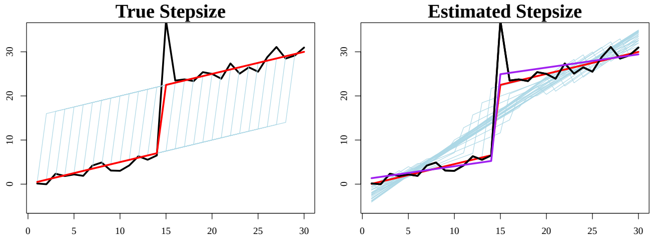
where $\mathbf{Z}$ is constructed as outlined before and

- $\Delta = diag(\delta^\gamma)$ is a selection matrix,
- $\delta^\gamma \in \{0, 1\}^{N(T-3)}$ are the selection indicators,
- $\gamma \in \mathbb{R}^{N(T-3)}$ are the coefficients of included breaks.

The name of the game is find each $\delta_{i,t}$ for which $\mathbb{E}(\delta_{i,t}|y) > P$

A Normal Mixture for ε

- **Need for outlier correction** to prevent bias in the step-estimation



- Gets uses indicator saturation to "dummy out" the outlier.
- We use a Normal mixture, both centered around zero, with an additional scaling parameter K in a data augmentation step:

$$\varepsilon_{i,t} \sim \pi_\varepsilon = \begin{cases} \mathcal{N}(\varepsilon_{i,t}; 0, \sigma^2) & \text{if } \delta^\varepsilon = 0 \\ \mathcal{N}(\varepsilon_{i,t}; 0, \sigma^2 K) & \text{if } \delta^\varepsilon = 1 \end{cases} \quad \text{for } K \gg 1$$

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Priors

We implement the model in a **4-step Gibbs sampler** procedure:

$$\sigma_i^2 \sim \mathcal{G}^{-1}\left(\frac{1}{100}, \frac{1}{100}\right)\Big\}\text{Error variance}$$

$$\beta \sim \mathcal{N}_p\left(0, \sigma^2 \lambda I_p\right)\Big\}\text{Covariate coefficients}$$

$$\left.\begin{array}{r} \eta \sim Beta(c_0, d_0) \\ \delta^{\varepsilon}|\eta \sim Bern(\eta) \end{array}\right\}\text{Outlier correction}^{1}$$

$$\left.\begin{array}{r} \delta_{i,t}^{\gamma}|\omega_i \sim Bern(\omega_i) \\ \gamma|\delta^{\gamma} \sim p(\vartheta_{\gamma}) \end{array}\right\}\text{Break detection}$$

1. Assume errors come from Gaussian mixture: $\tilde{\varepsilon} = (1 - \delta^{\varepsilon})\,\hat{\varepsilon} + \delta^{\varepsilon}\,\hat{\varepsilon}/K^{1/2}$ and use them in data augmentation step before further estimation.

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Priors

$$\gamma \mid \delta^{\gamma}, \cdot \sim p(\vartheta_{\gamma})$$

- Dispersion parameter ϑ_{γ} is key due to information paradox.
- We use non-local priors (NLPs) proposed for model selection and Bayesian testing in Johnson and Rossell (2010, 2012).
  - A priori **parameter independence** across time and observations.
  - In particular, the **Inverse Moment Prior (iMom)** is convenient as it allows for model consistency for $p = \mathcal{O}(T)$

Non Local Priors

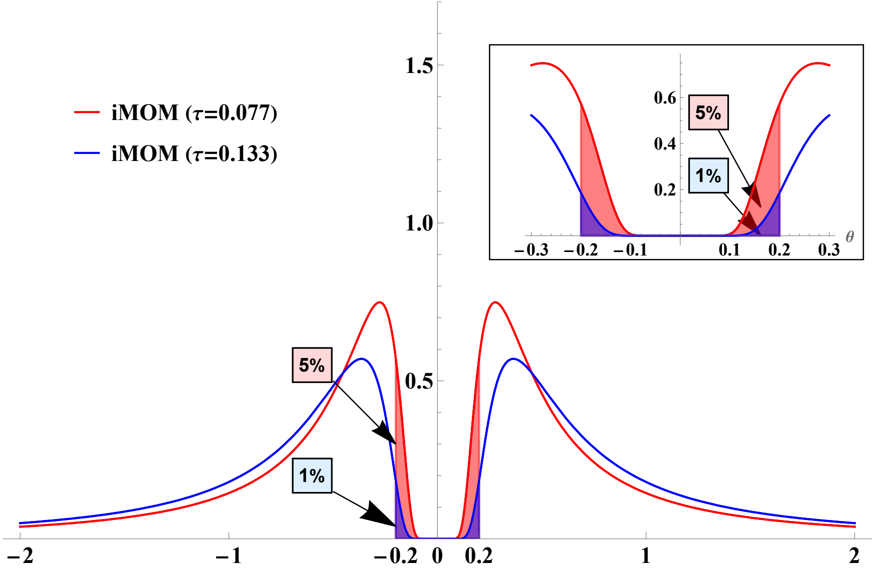
- **iMOM prior density** for some $k, \nu, \tau > 0$ takes the form:

$$\pi_i(\gamma|\gamma_0, k, \nu, \tau) = \frac{k\tau^{\nu/2}}{\Gamma(\nu/2k)} \left((\gamma - \gamma_0)^2\right)^{-(\nu+1)/2} \exp\left\{-\left(\frac{(\gamma-\gamma_0)^2}{\tau}\right)^{-k}\right\}$$

- **Standard-parameterization:**
  - $\gamma_0 = 0$
  - $k = 1$
  - $\nu = 1$ (Cauchy tails)

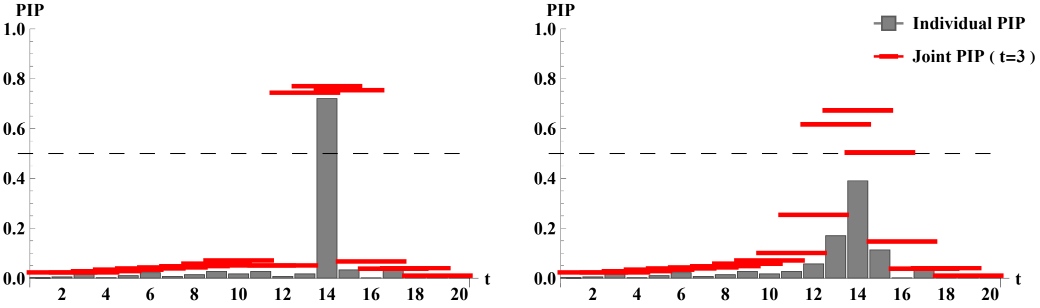
- τ **controls the a-priori probability of breaks with a given size.**

Non Local Priors: τ calibration



Break Uncertainty

- Probabilistic setup allows for a **measure of break uncertainty**
- **Naturally nested in the model** using MCMC draws of $\delta_{i,t}$



- Combination of individual breaks via $P(\delta_{i,t} = 1 \vee \delta_{i,t+l} = 1)$ possible

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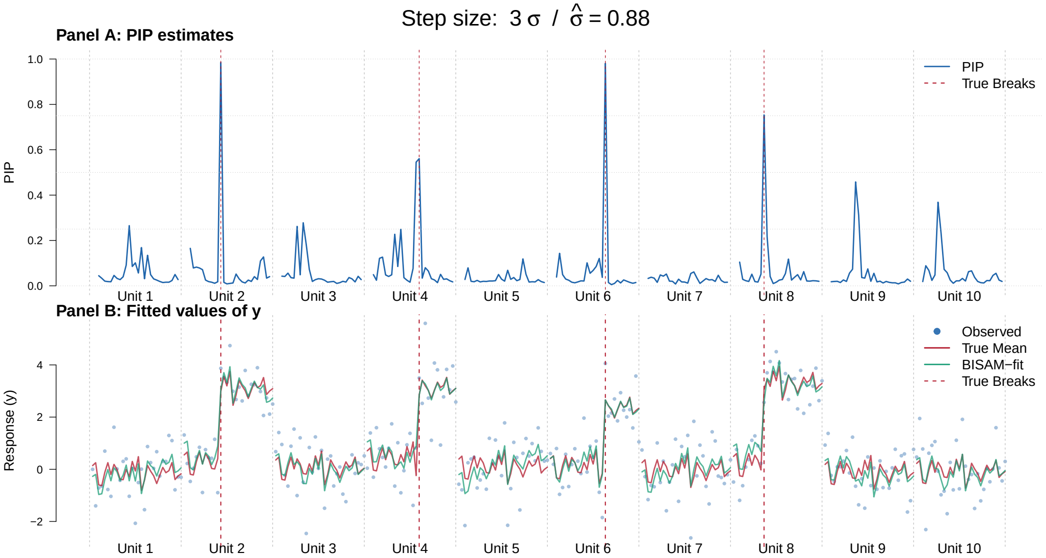
Simulation Study

We use simulated data to compare our approach to “Gets”

- N = 10
- T = 30
- Break size = {1, 1.5, 2, 3, 6, 10} $\times$ SD of $\varepsilon_{i,t}$
- Two settings:
  1. Sparse: 4 units with 1 break
  2. Dense: 8 units, 4 of which have 2 breaks
- # of repetitions = 100
- α-level of Gets set to 0.05, $\tau \approx 2$ s.t. $P(|\gamma| \leq SD(\varepsilon)|\tau) = 0.05$

Simulation Study



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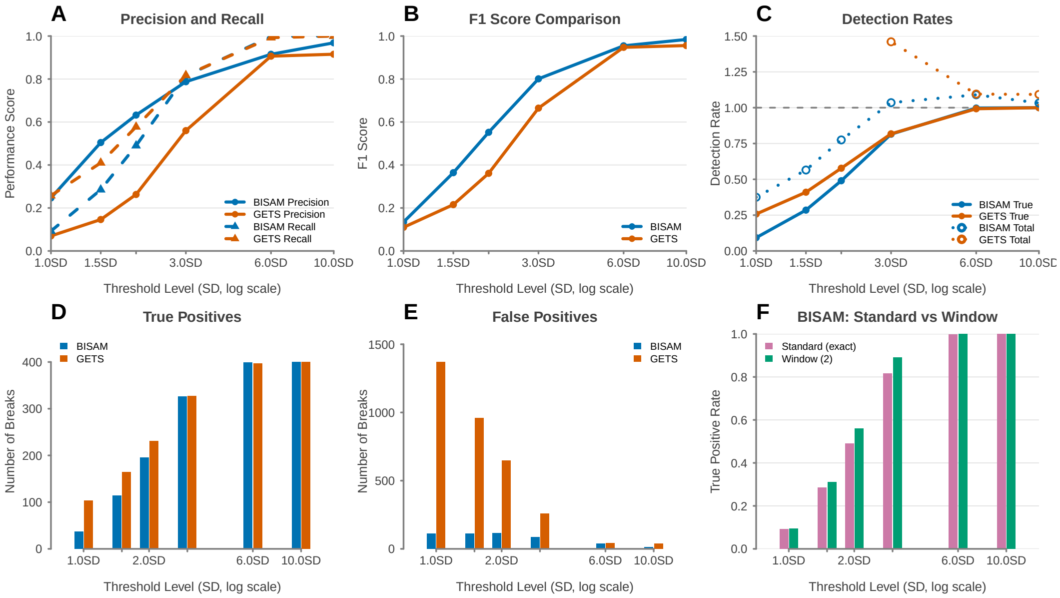

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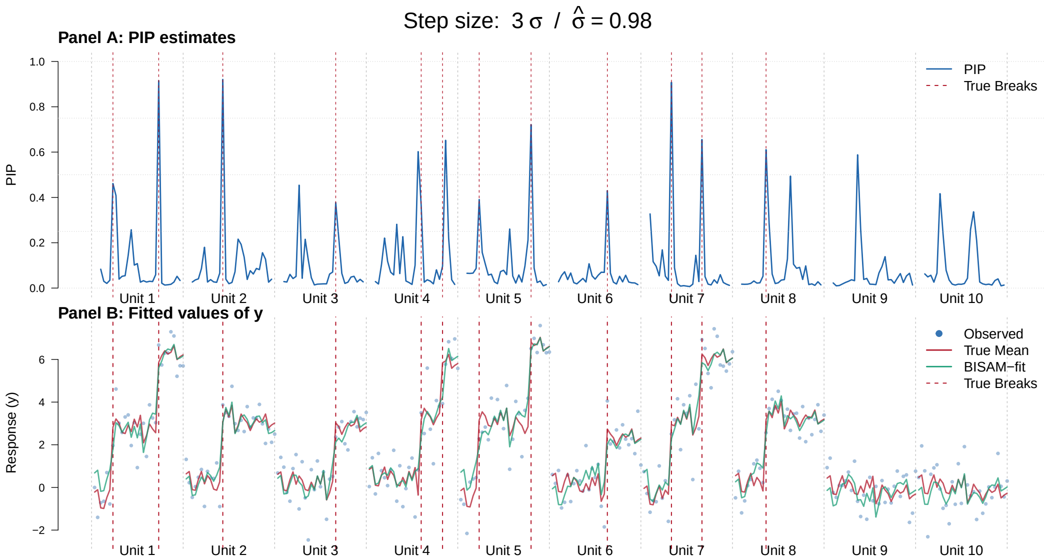

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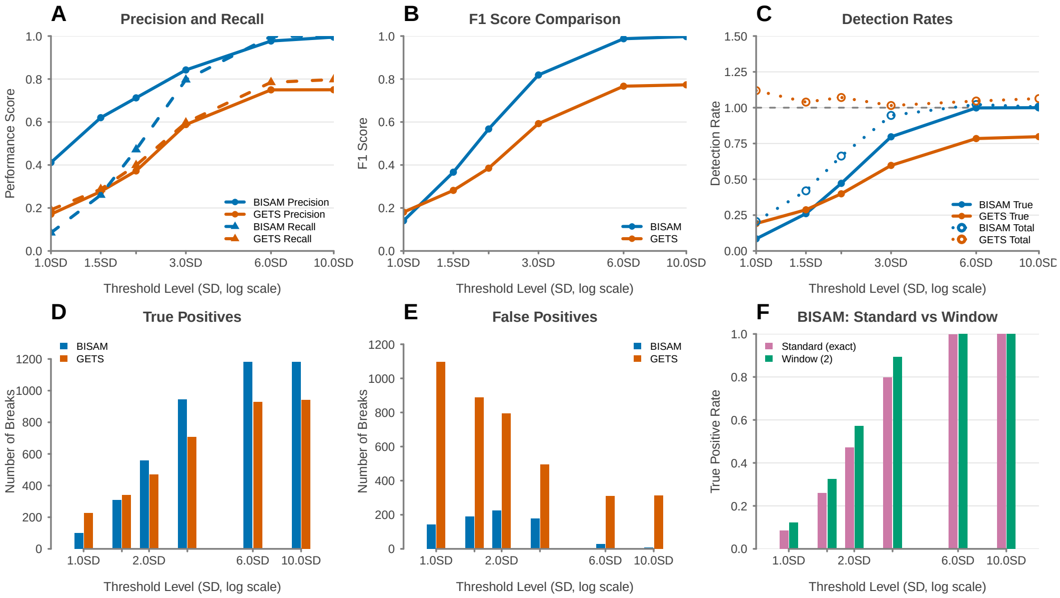

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Detecting Breaks in Emission Data

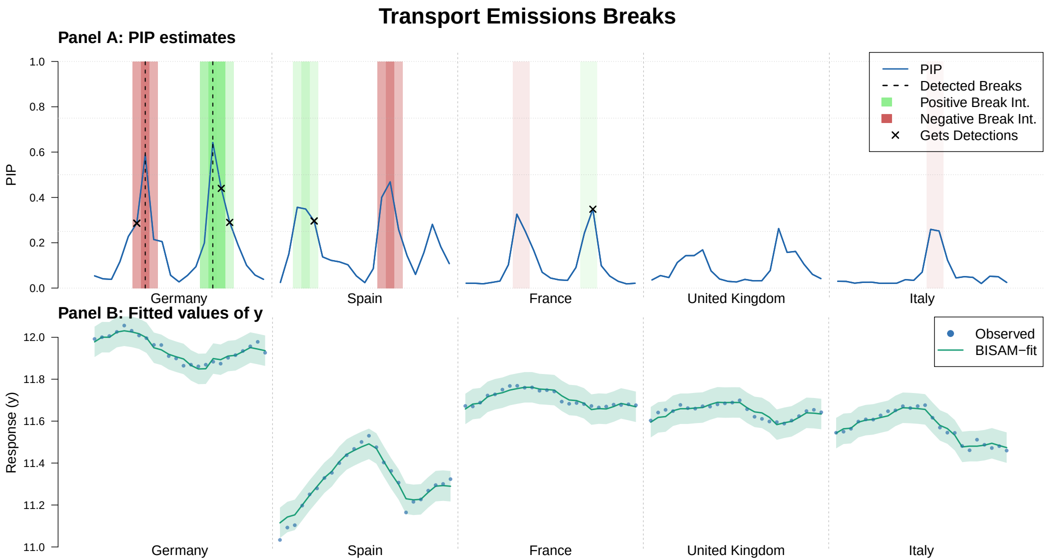
For comparison, we replicate Koch et al. (2022) that search for **breaks in transport emissions data** and assign them to **climate policies** (or mixes).

- **Dependent variable:** (log) transport emissions
- **Controls:** (log) GDP, GDP^2, POP + two-way FEs
- **Region:** EU15 countries
- **Time-span:** 1995-2018 (yearly)

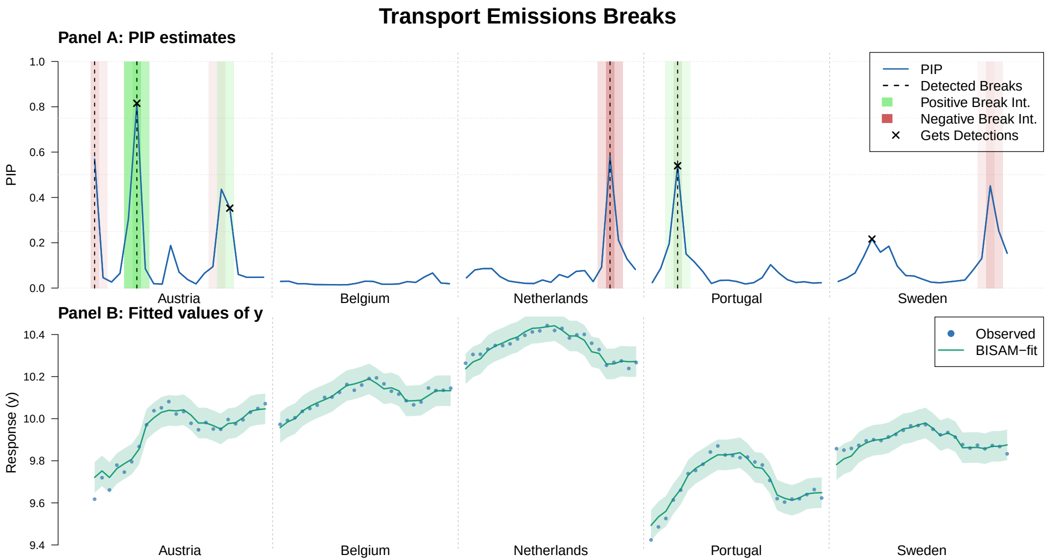
$$log(CO_2) = FE\alpha + log(GDP)\beta_1 + log(GDP^2)\beta_2 + log(POP)\beta_3 + \mathbf{Z}\Delta\gamma + \varepsilon$$

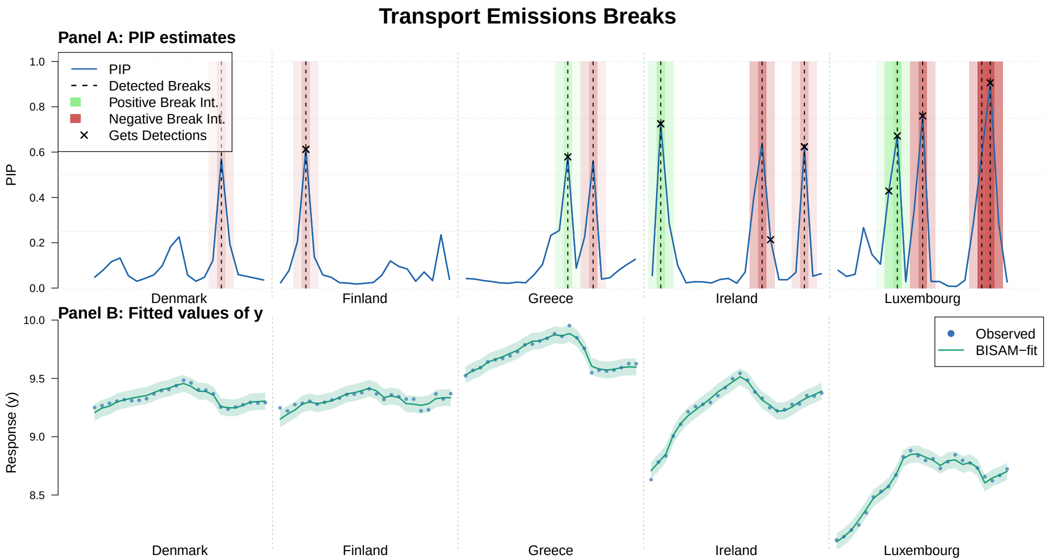
- We set $\tau \approx 2$ s.t. $P(|\gamma| \leq SD(\varepsilon)|\tau) = 0.05$

Replication: Results 1/3



Replication: Results 2/3




Transport Emissions Breaks
Panel A: PIP estimates
PIP
Detected Breaks
Positive Break Int.
Negative Break Int.
Gets Detections
1.0
0.8
0.6
0.4
0.2
0.0
Denmark
Finland
Greece
Ireland
Luxembourg
Panel B: Fitted values of y
Response (y)
10.0
9.5
9.0
8.5
Observed
BISAM−fit
Denmark
Finland
Greece
Ireland
Luxembourg

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Application: Mining Segmentation

- As an environmental application, we look at **Toka Tindung mine** in Indonesia and aim to track its expansion over time
- Use peak **Enhanced Vegetation Index (EVI)** index for 90×90m pixels within pre-defined polygon (Sepin, Vashold, and Kuschnig 2025)
- Satellite data from **LANDSAT8** for $N \approx 2000$ and $T = 22$
- Utilizes different parts of model:
  - FE estimates for determination of always/never-treated units
  - Breaks for transitions from forest to mine
  - Outlier detection to mitigate double-counting and cloud artifacts
  - Windows for break-time uncertainty and more laissez-faire detection

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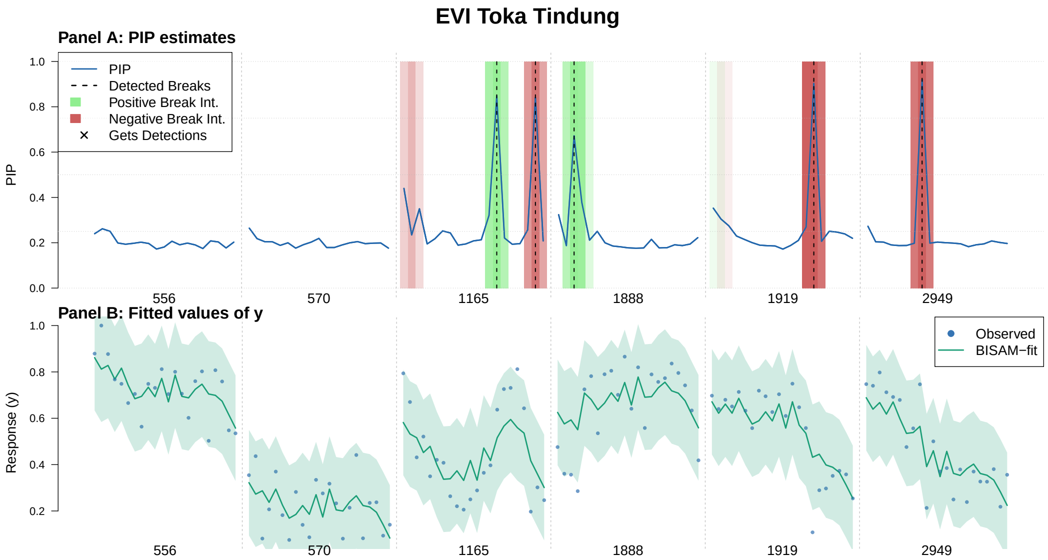

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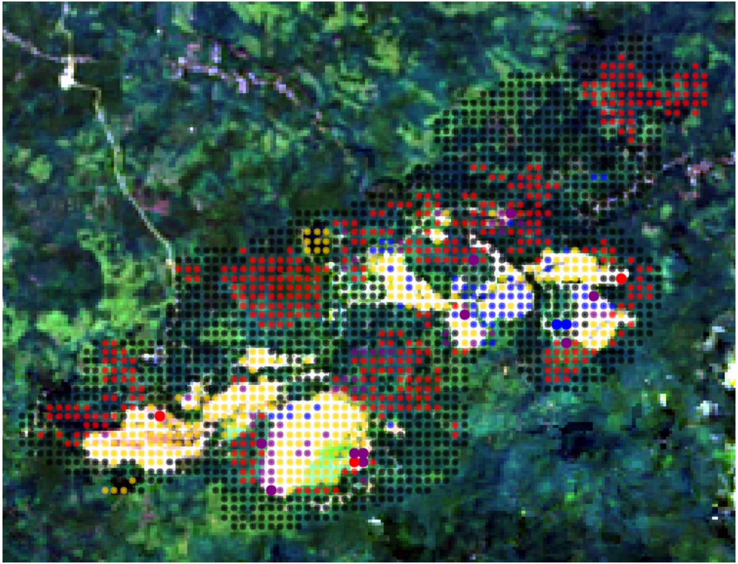

Figure: Toka Tindung in H1 2015

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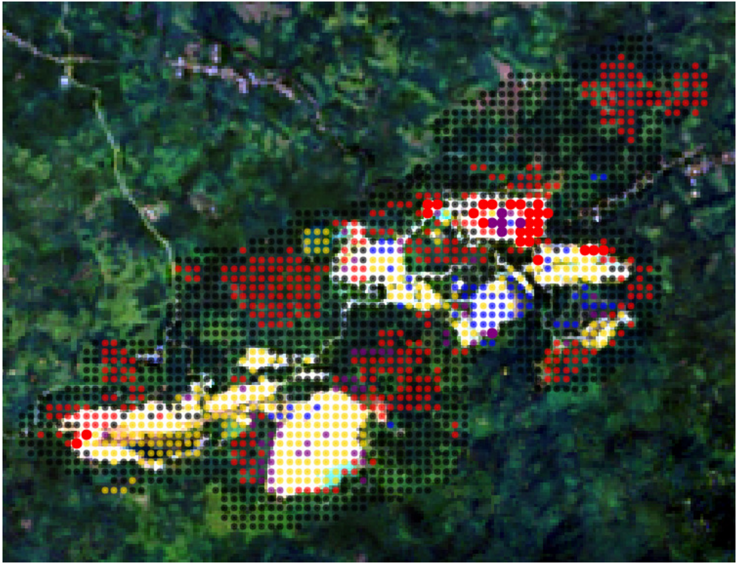

Figure: Toka Tindung in H2 2017

Application: Mining Segmentation

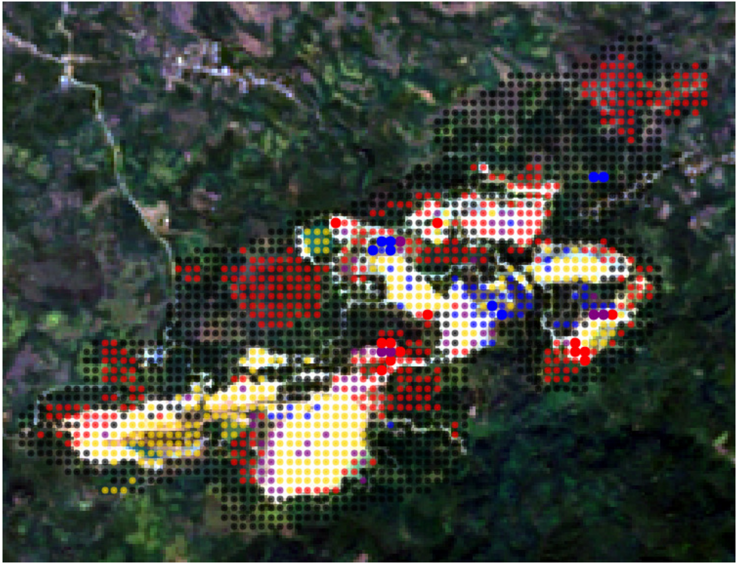

Figure: Toka Tindung in H2 2019

Application: Mining Segmentation

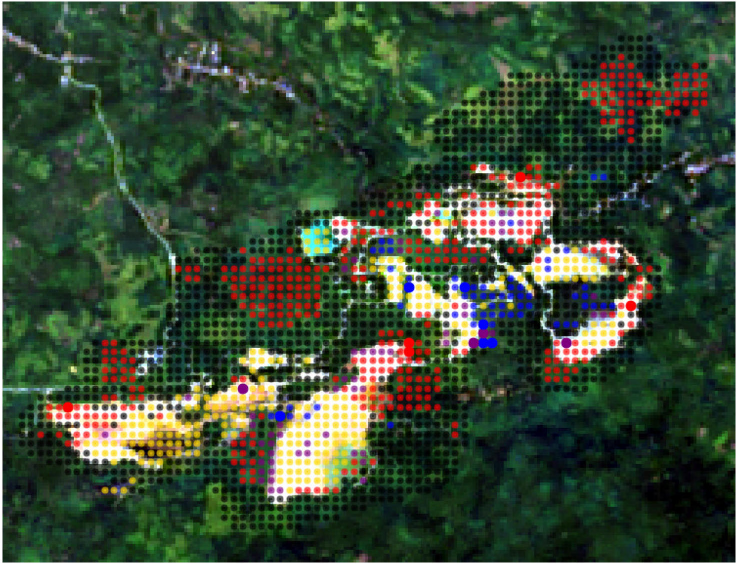

Figure: Toka Tindung in H2 2020

Application: Mining Segmentation

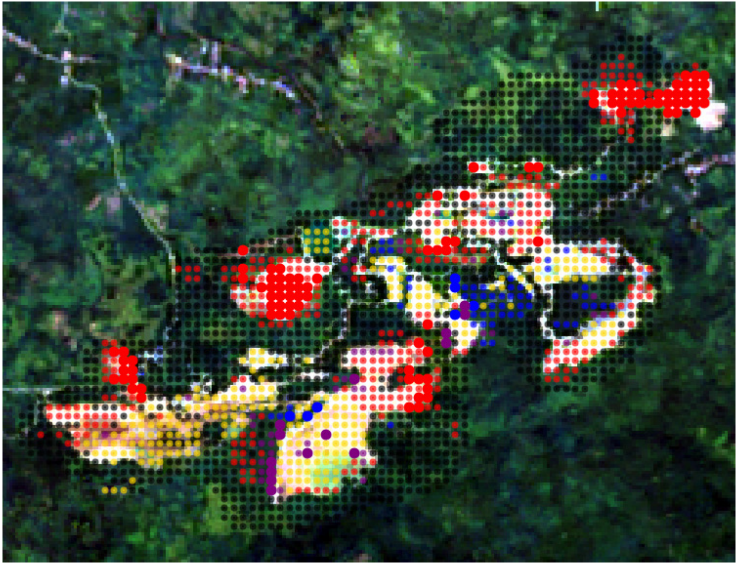

Figure: Toka Tindung in H2 2021

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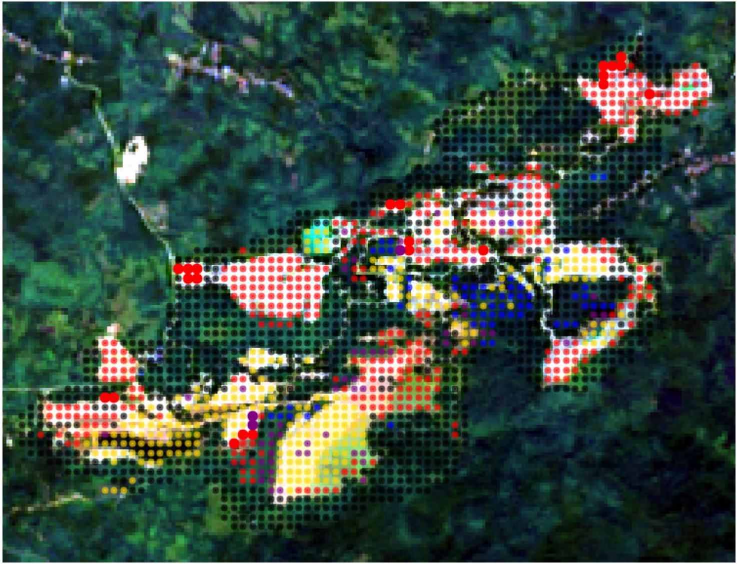

Figure: Toka Tindung in H1 2023

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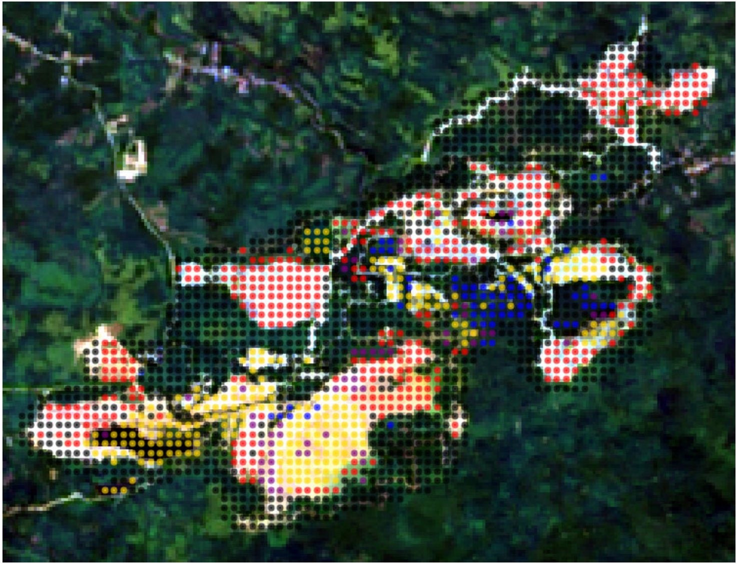

Figure: Toka Tindung in H2 2024

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Summary & Caveats

PRO:

A very flexible break detection model with:

- **Interpretable prior parameters**
- Competitive **detection quality**
- **Break uncertainty** quantification

CONS:

- The model **scales well with** N **but less with** T
- **Computational complexity is considerable** (but manageable)
- The **iMom prior is not conjugate**
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